Universität Bremen

Institute for Artificial Intelligence

Faculty 03

Mathematics &

Computer Science

Robot Programming with ROS

2. Coordinates & Transforms

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26th Oct. 2023

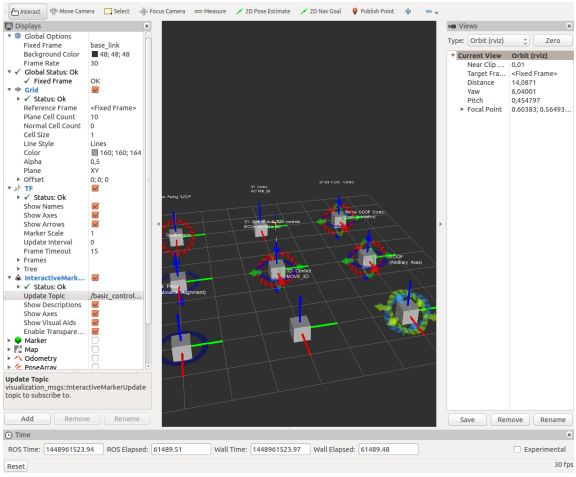
Intuition

```
$ roscore
$ rosrun interactive_marker_tutorials basic_controls
$ rosrun rviz rviz
```

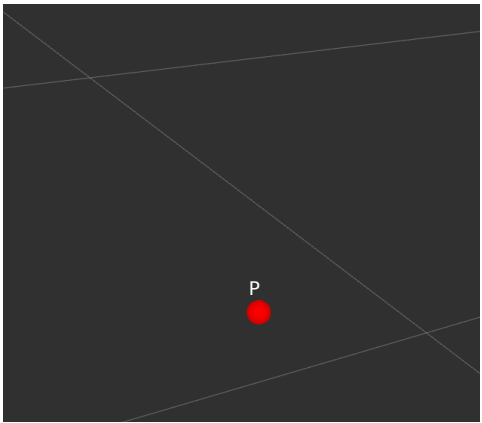
Fixed Frame: `base_link` >

Add by topic:

`InteractiveMarker`

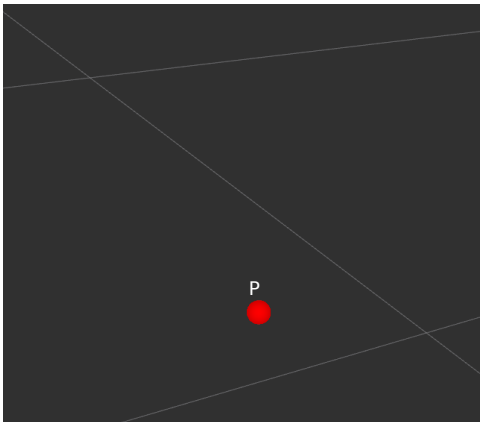


3D Geometry Basics



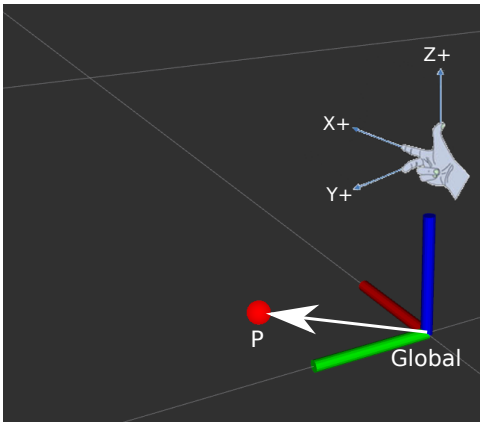
- What is a point in space? How do we represent it?

3D Geometry Basics



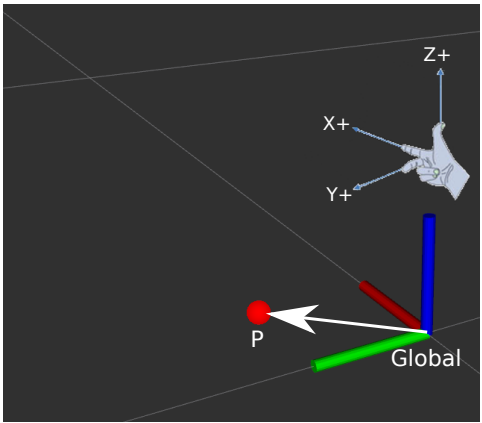
- What is a point in space? How do we represent it?
- Cartesian coordinates (x, y, z)

3D Geometry Basics



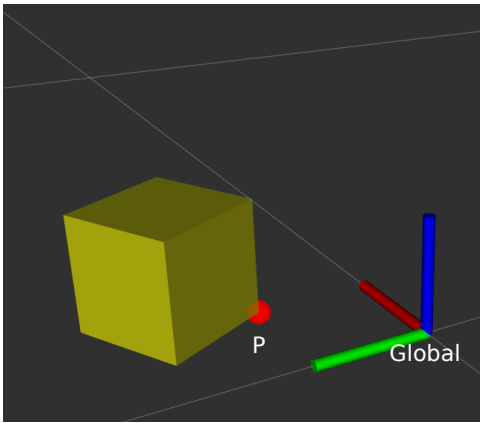
- What is a point in space? How do we represent it?
- Cartesian coordinates (x, y, z)
- Reference frame
  ${}_{global}P = (0.1, 0.1, 0.0)$

3D Geometry Basics



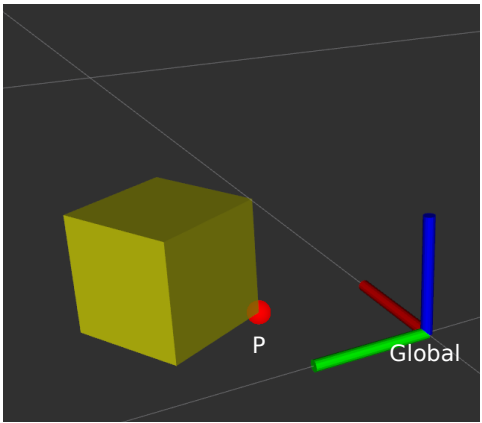
- What is a point in space? How do we represent it?
- Cartesian coordinates (x, y, z)
- Reference frame
  ${}_{global}P = (0.1, 0.1, 0.0)$
- Right-hand rule:
  $(X, Y, Z) \rightarrow (R, G, B)$

3D Geometry Basics



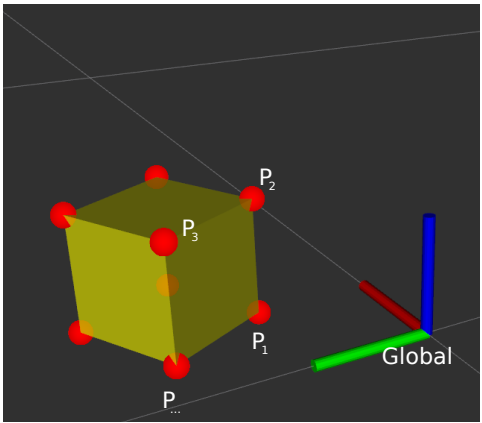
- How do we represent an object in 3D?

3D Geometry Basics



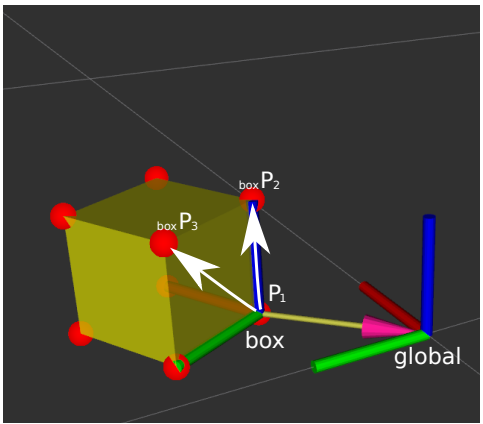
- How do we represent an object in 3D?
- What is an object?

3D Geometry Basics



- How do we represent an object in 3D?
- What is an object?
- Problem: all vertices change coordinates during movement

3D Geometry Basics

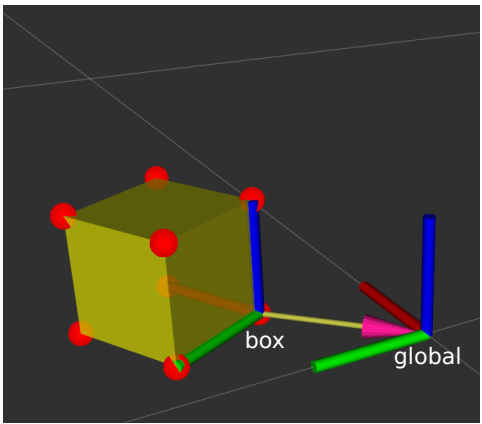


- How do we represent an object in 3D?
- What is an object?
- Problem: all vertices change coordinates during movement
- Solution: describe points on object relative to an object frame

  ${}_{global}P_1 = (0.1, 0.1, 0.0)$

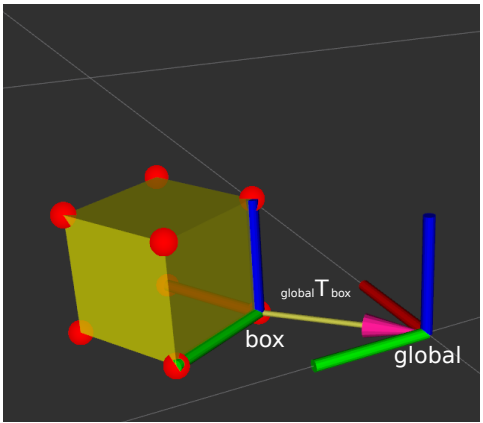
  ${}_{box}P_1 = (0.0, 0.0, 0.0)$

3D Geometry Basics



- How do we represent an object in 3D?
- What is an object?
- Problem: all vertices change coordinates during movement
- Solution: describe points on object relative to an object frame
  ${}_{global}P_1 = (0.1, 0.1, 0.0)$
  ${}_{box}P_1 = (0.0, 0.0, 0.0)$
- What do we need to describe the object frame?

3D Geometry Basics



- *box* has a position and orientation relative to *global*
- *position* & *orientation* together are called *pose*
- ${}_{global}T_{box}$ is a transformation that transforms poses from *box* to *global*
- How do we represent position and orientation?

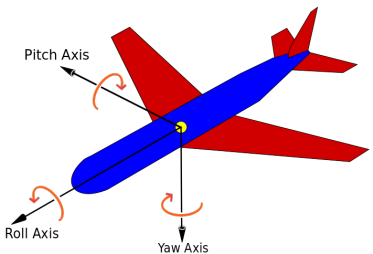
Rotation Representations

There are 4 common ways to describe rotations:

- euler angles
- rotation matrix
- axis-angle
- quaternion

Euler Angles

- Describes orientation using 3 angles:
  roll (x-rotation), pitch (y-rotation), yaw (z-rotation)
- Rotations are applied in sequence.
  What is the sequence is defined through a convention.
  There are many conventions, most common are z-y-x, x-y-z and z-x-z



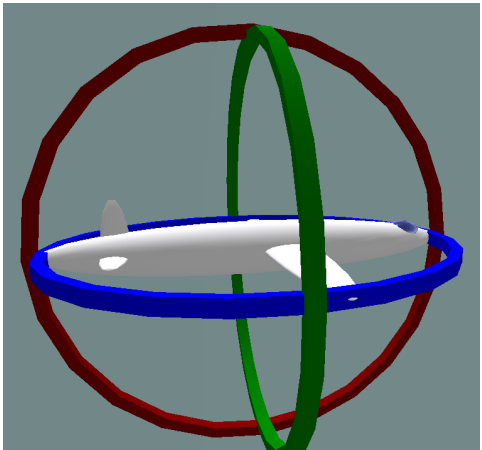
Euler Angles

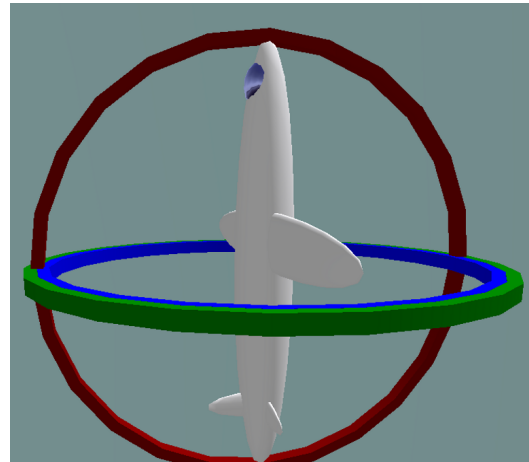
- \+ easy to interpret
- – has a Gimbal lock problem
- – not suited for interpolation
- – there are many possible conventions, always make sure you know which one is used!

→ only useful for user interaction

Euler Angles

Loss of one degree of freedom, e.g. after 90^o pitch (in this case red axis).



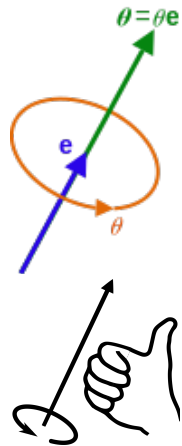


Axis-Angle

- any rotation can be represented as right hand rotation by θ degree about a unit vector e
- angle can be encoded in length of the vector

$$\begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix}, \theta \rightarrow \begin{pmatrix} \theta e_x \\ \theta e_y \\ \theta e_z \end{pmatrix}$$

- can be rotated by rotation matrices using matrix multiplication



Axis-Angle

- – math can get unstable when θ is close to 0 or π, because there are infinitively many possible axis
- – represents rotation by θ differently from $\theta + 2\pi$, but it is the same rotation
- \+ easy interpolation, just scale the angle, but take into account that $\theta = \theta + 2\pi$
- → more useful when describing rotation differences/changes instead of orientations, found in ROS messages like Twist or Accel.

Quaternion

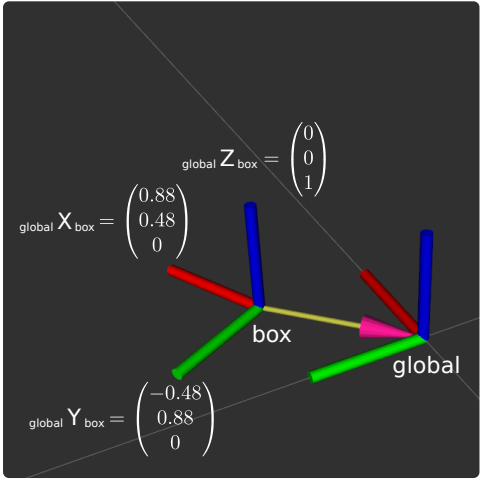
- $q = (x, y, z, w)$
- number system introduced by Hamilton as an extension of complex numbers, only use case is representation of rotations
- only unit quaternions are used to represent rotations
- can be interpreted as an improved version of axis-angle
- $$\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}, \alpha \rightarrow \begin{pmatrix} a_x \cdot \sin(\alpha/2) \\ a_y \cdot \sin(\alpha/2) \\ a_z \cdot \sin(\alpha/2) \\ \cos(\alpha/2) \end{pmatrix}$$

Rotation Matrix

- 3 x 3 matrix R
- is an orthogonal matrix, i.e. $det(R) = 1$ and $R^{-1} = R^T$
- this means, all raw (and correspondingly column) vectors are unit vectors, orthogonal to each other
- example: $R = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ rotates about z-axis by θ

Rotation Matrix

- example:
  $$R = \begin{pmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
  rotates about z-axis by θ
- $${}_{global}R_{box} = \begin{pmatrix} 0.88 & -0.48 & 0 \\ 0.48 & 0.88 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
- columns are axis of `box` in the `global` coordinate frame



Rotation Matrix

+ easiest to do math with
  - rotate a vector with rotation matrix using matrix multiplication
  - rotation matrices can be combined using matrix multiplication
+ easy to construct rotation matrix from 3 vectors
+ can be extended to include translation in 4x4 matrix
– uses 9 numbers to describe 3 degrees of freedom
– matrix operations result in buildup of rounding error, you might have to normalize often
– not suitable for interpolation

Quaternion

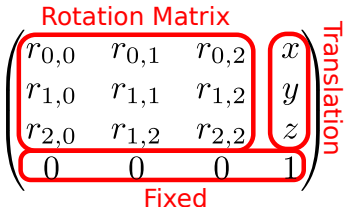
+ in contrast to axis-angle, stable when angle is close to zero and π
+ removes the $\theta = \theta + 2\pi$ problem from axis-angle
+ more compact representation than rotation matrices
+ best for interpolation (slerp algorithm)
– difficult to interpret
→ most useful for interpolation and describing orientations
ROS standard for representing poses

Rotations representations

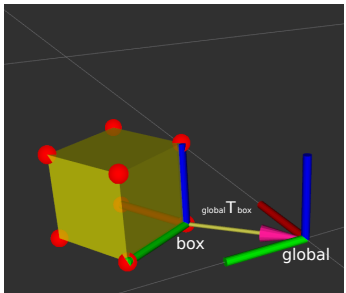
- use euler angles only on an interface level
- use axis-angle or quaternion for rigid body dynamics
- use quaternions when storing/sending orientation information or for interpolation
- else use rotation matrices for easy mathematical operations

Homogeneous Transformations

- 4 x 4 matrix to represent pose transformations
- ${}_aT_b$ means transform from frame *b* to *a*, i.e.:
  ${}_aT_b \cdot {}_bP = {}_aP$
- ${}_aT_b$ is the same as ${}_aP_b$, i.e. pose of origin of *b* in *a*
- combined transformation:
  ${}_cT_b \cdot {}_bT_a = {}_cT_a$
- invertible: ${}_bT_a^{-1} = {}_aT_b$
- but ${}_bT_a^{-1} \neq {}_bT_a^T$



$$\begin{pmatrix} r_{0,0} & r_{0,1} & r_{0,2} & x \\ r_{1,0} & r_{1,1} & r_{1,2} & y \\ r_{2,0} & r_{1,2} & r_{2,2} & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

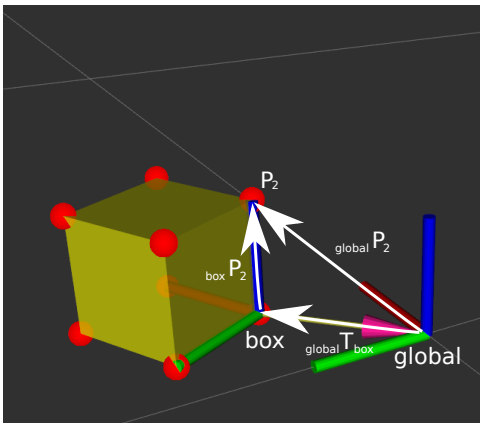


Homogeneous Transformation

- How do we do ${}_cT_b \cdot {}_bP = {}_cP$?
- Append 1 to point P, before matrix multiplication:

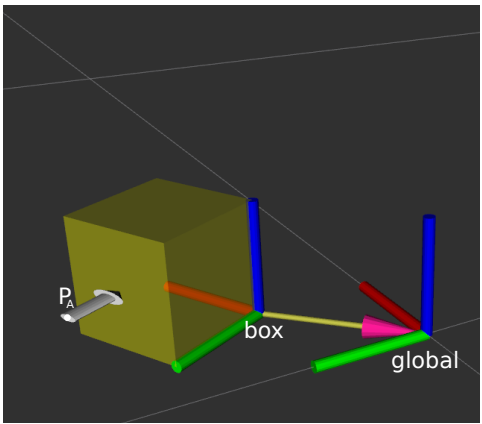
$$\begin{pmatrix} r_{0,0} & r_{0,1} & r_{0,2} & x \\ r_{1,0} & r_{1,1} & r_{1,2} & y \\ r_{2,0} & r_{2,1} & r_{2,2} & z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} p_x \\ p_y \\ p_z \\ 1 \end{pmatrix} = \begin{pmatrix} r_{0,0}p_x + r_{0,1}p_y + r_{0,2}p_z + x \cdot 1 \\ r_{1,0}p_x + r_{1,1}p_y + r_{1,2}p_z + y \cdot 1 \\ r_{2,0}p_x + r_{2,1}p_y + r_{2,2}p_z + z \cdot 1 \\ 0p_x + 0p_y + 0p_z + 1 \cdot 1 \end{pmatrix}$$

Homogeneous Transformation



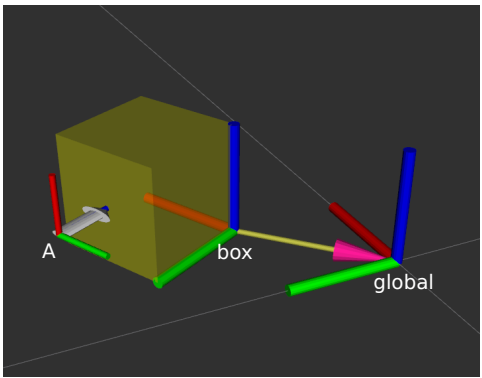
- to transform ${}_{box}P_2$ into the global frame ${}_{global}P_2$, multiply with ${}_{global}T_{box}$
- ${}_{global}P_2 = {}_{global}T_{box} \cdot {}_{box}P_2$

Homogeneous Transformation



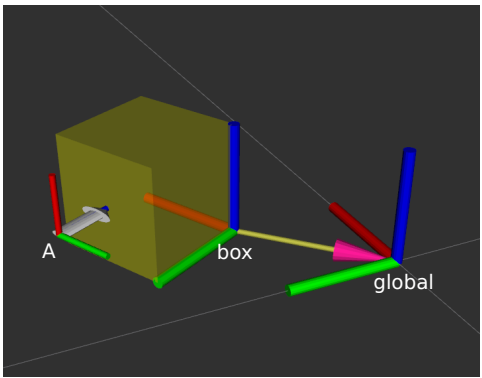
- what is the pose of P_A in global coordinate frame: ${}_{global}P_A$?
- choose frame where it is the easiest to express a pose
- ${}_{box}P_A = (0.05, 0.15, 0.05, 1.0)$
- ${}_{global}P_A = {}_{global}T_{box} \cdot {}_{box}P_A$

Homogeneous Transformation



$$ {}_{box}T_A = \begin{pmatrix} & & & 0.05 \\ & & & 0.15 \\ & & & 0.05 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Homogeneous Transformation



$$ {}_{box}T_A = \begin{pmatrix} 0 & -1 & 0 & 0.05 \\ 0 & 0 & -1 & 0.15 \\ 1 & 0 & 0 & 0.05 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

Points in ROS

Point in 3D: *{x, y, z}*

Point

```python
from geometry_msgs.msg import Vector3

point = Vector3(1, 2, 0)
print(point)   # x: 1, y: 2, z: 0
print(point.y) # 2
```

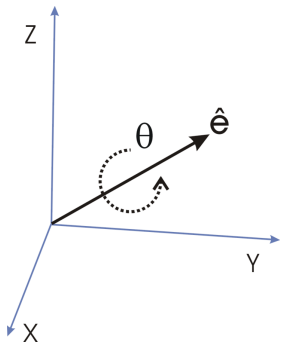
Object in 3D: *{position, orientation}*

- Position: *{x, y, z}*
- Orientation: axis-angle / rotation matrix / quaternions / ...

Rotations in ROS

Axis-Angle representation:

$$< axis, angle >= \left\langle \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \theta \right\rangle$$



Axis-Angle → Quaternion:

$$Q = \begin{pmatrix} q_x \\ q_y \\ q_z \\ q_w \end{pmatrix} = \begin{pmatrix} a_x \sin(\theta/2) \\ a_y \sin(\theta/2) \\ a_z \sin(\theta/2) \\ \cos(\theta/2) \end{pmatrix}$$

Quaternion

```
1  from geometry_msgs.msg import Quaternion
2
3  quaternion = Quaternion(0, 0, 0, 1)
4  print(quaternion) # x: 0, y: 0, z: 0, w: 1
```

Poses in ROS

Pose Matrix

```python
from geometry_msgs.msg import Pose

pose = Pose(point, quaternion)
print(pose)
# position:
# x: 1
# y: 2
# z: 0
# orientation:
# x: 0
# y: 0
# z: 0
# w: 1
```

Transformations in ROS

Identity Matrix

```
1 import tf.transformations as tr
2 I = tr.identity_matrix()
3 #[[1. 0. 0. 0.]
4 # [0. 1. 0. 0.]
5 # [0. 0. 1. 0.]
6 # [0. 0. 0. 1.]]
```

Docs: `https://docs.ros.org/en/jade/api/tf/html/python/transformations.html`

Transformations in ROS

Translation + Rotation

```
1  import numpy
2  Rz = tr.rotation_matrix(numpy.pi*0.5, (0, 0, 1))
3  Vx = tr.translation_matrix([2, 0, 0])
4  Vx_Rz = tr.concatenate_matrices(Vx, Rz)
5  print(Vx_Rz)
6  #[[0. -1.  0.  2.]
7  # [1.  0.  0.  0.]
8  # [0.  0.  1.  0.]
9  # [0.  0.  0.  1.]]
```

Docs: `https://docs.ros.org/en/jade/api/tf/html/python/transformations.html`

Transformations in ROS

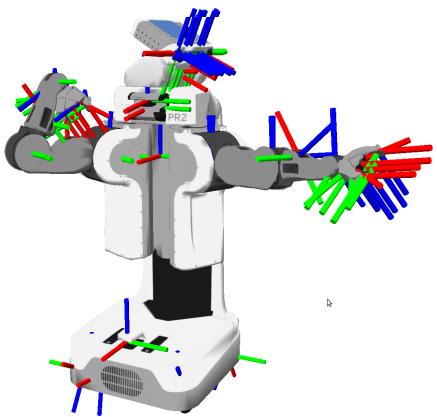
Inverse

```
1  tr.inverse_matrix(Vx_Rz)
2  #[[ 0.  1.  0.  0.]
3  # [-1.  0.  0.  2.]
4  # [ 0.  0.  1.  0.]
5  # [ 0.  0.  0.  1.]]
```

Docs: `https://docs.ros.org/en/jade/api/tf/html/python/transformations.html`

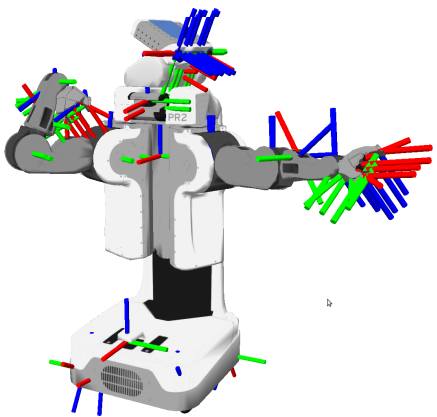
Plan

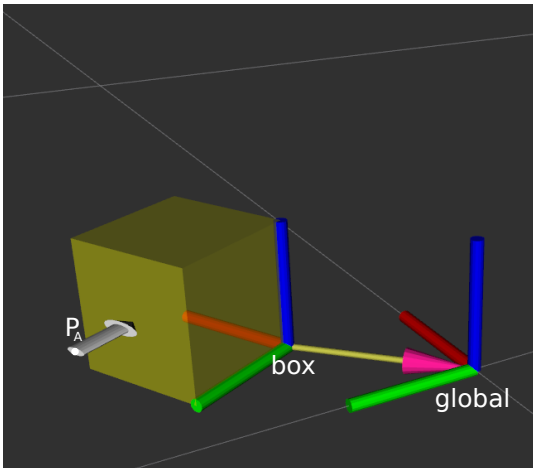
Motivation



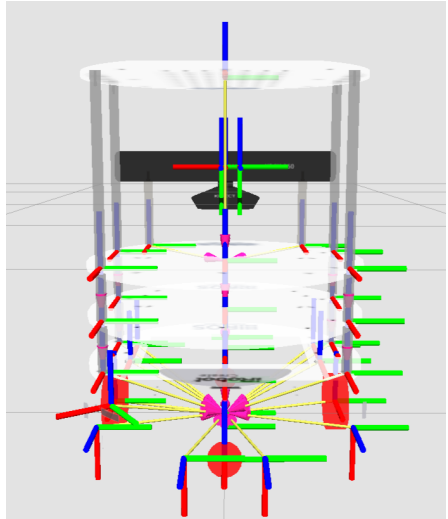
- Robots consist of many *parts* aka *links*
- Each link has its own *coordinate frame*
- Links change their position over time (including the robot base)
- Sensors measurements are defined in their own frame
- Example: transformations from camera to hand coordinates are needed for grasping objects

Motivation





TurtleBot Coordinate Frames



Tracking Coordinate Frame Changes

- Transforms are produced by different nodes:
  - Localization node (AMCL, gmapping) for finding robot's pose in map
  - Odometry node (base driver) for tracking movement since initial pose
  - Joint positions (robot controllers and robot_state_publisher)
- Many publishers, many consumers
- Distributed system, redundancy issues, ...

↓

- **TF**: a coordinate frame tracking system

What is tf?

trans**f**orm Library – a distributed coordinate frame tracking system

- Standardized protocol for publishing transforms to tf listeners
- Looking up and calculating transforms by asking tf listeners
- tf listener can be either local Lisp program or global tf buffer
- default global tf buffer is TF2's buffer_server
- ROS API for looking up, calculating and sending transforms
- Transforms are published on `/tf` and `/tf_static` topics:

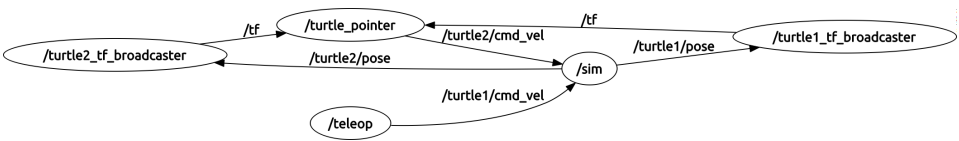
  `/tf`
  - for all transforms that change over time
  - publish with a fixed rate, even if transform didn't change

  `/tf_static`
  - assumed to be static, thus never outdated
  - useful for reducing redundancy
  - only publish once with latched flag

TurtleSim TF

Launch the turtlesim TF demo:

```
$ roslaunch turtle_tf turtle_tf_demo.launch
```



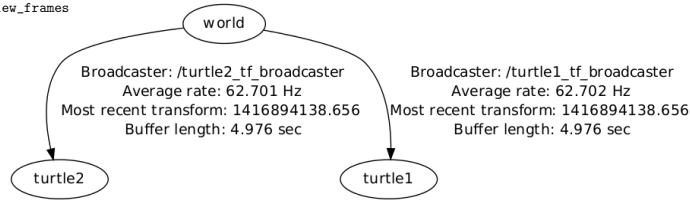
Utilities

- `view_frames`
- `tf_echo`
- `tf_monitor`
- `static_transform_publisher`
- RViz

Utilities

Generate a TF tree graph:

```
$ rosrun tf view_frames
```



- TF tree consists of frames (links) and the transforms between them.
- Each transform is cached (10 secs default caching time)
- Transforms must form a proper tree (no cycles)
- Can have disconnected trees, but you can only ask for transforms inside of the same tree

Utilities

```
$ rosrun tf tf_echo <source_frame> <target_frame>
```

tf_echo

```
1  $ rosrun tf tf_echo turtle1 turtle2
2  At time 0.000
3  - Translation: [0.100, 0.100, 0.000]
4  - Rotation: in Quaternion [0.000, 0.000, 0.247, 0.969]
5              in RPY (radian) [0.000, -0.000, 0.500]
6              in RPY (degree) [0.000, -0.000, 28.648]
```

Utilities

- `rosrun tf2_ros static_transform_publisher x y z yaw pitch roll frame_id child_frame_id`
  or
  `rosrun tf2_ros static_transform_publisher x y z qx qy qz qw frame_id child_frame_id`
- publishes ${}_{global}T_{box}$

static_transform_publisher

```
$ rosrun tf2_ros static_transform_publisher 0.1 0.1 0 3.14 0 0 global box
```

Utilities

- `rosrun tf tf_monitor`

tf_monitor

```
$ rosrun tf tf_monitor
RESULTS: for all Frames

Frames:
Frame: turtle1 published by /turtle1_tf_broadcaster Average Delay: 0.000382455 Max Delay: 0...
Frame: turtle2 published by /turtle2_tf_broadcaster Average Delay: 0.000267847 Max Delay: 0...

All Broadcasters:
Node: /turtle1_tf_broadcaster 64.6996 Hz, Average Delay: 0.000382455 Max Delay: 0.000991178
Node: /turtle2_tf_broadcaster 64.7127 Hz, Average Delay: 0.000267847 Max Delay: 0.00133464
```

TF data types

- `frame_id`: name of the published frame
- `child_frame_id` has to be an existing frame
- `stamp`: time when this transform is valid
- *child_frame_id* T *frame_id*

tf2_msgs/TFMessage

```
geometry_msgs/TransformStamped[] transforms
 std_msgs/Header header
  uint32 seq
  time stamp
  string frame_id
 string child_frame_id
 geometry_msgs/Transform transform
  geometry_msgs/Vector3 translation
   float64 x
   float64 y
   float64 z
  geometry_msgs/Quaternion rotation
   float64 x
   float64 y
   float64 z
   float64 w
```

TF and time

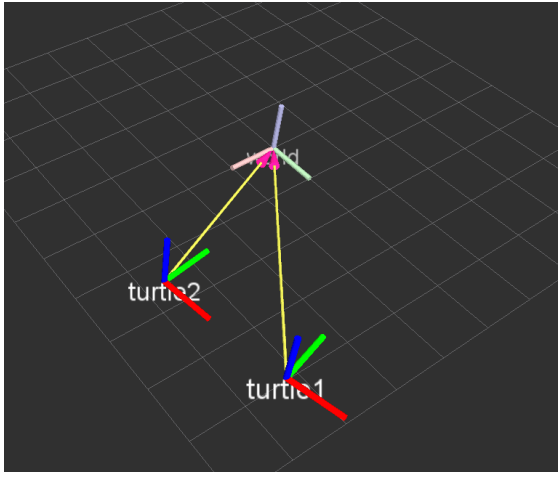
- tf buffers transforms for X seconds
- possible to lookup transforms from the past
- tf interpolates frames
- tf does not extrapolate! it can't see into the future

Python TF

tf

```python
import rospy
import tf
rospy.init_node('turtle_tf_listener')
listener = tf.TransformListener()
rospy.wait_for_service('spawn')
rate = rospy.Rate(10.0)
while not rospy.is_shutdown():
    try:
        (trans,rot) = listener.lookupTransform('/turtle2', '/turtle1', rospy.Time(0))
        print(trans, rot)
    except (tf.LookupException, tf.ConnectivityException, tf.ExtrapolationException):
        print("No frame found")
        pass
    rate.sleep()
```

$ rosrun rviz rviz



Links

- Gilbert Strang's MIT course on linear algebra (free access):
  `https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/`
- 3Blue1Brown's essence of linear algebra (Playlist):
  `https://www.youtube.com/watch?v=fNk_zzaMoSs&list=PLZHQObOWTQDPD3MizzM2xVFitgF8hE_ab`
- Writing a TF Listener:
  `https://wiki.ros.org/tf/Tutorials/Writing%20a%20tf%20listener%20%28Python%29`
- Documentation of the tf.transformations package:
  `https://docs.ros.org/en/jade/api/tf/html/python/transformations.html`

Plan

Assignment 2

- Update repository:
  `$ git pull origin`
- Check the new assignment:
  `https://github.com/artnie/rpwr-assignments`
- Solve in the Binder:
  `https://binder.intel4coro.de/v2/gh/artnie/pycram/binder`

Info

- Assignment points: 8 points
- Due: 01.11.23
- Next class: 02.11.23, 14:00

Q & A

Thanks for your attention!